SKYLAB DOCKING RESPONSE ANALYSIS

By

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ABSTRACT

The salient features of the mathematical formulation and digital algorithm developed by Martin-Marietta Corporation, Denver Division for the Skylab docking response analysis are presented. The mathematical model considers the docking bodies to be in proximity such that collision is imminent. The coupling of the docking mechanism and the elastic responses of the vehicles are included. The Apollo probe/drogue docking system is considered.

INTRODUCTION

The docking maneuver and its associated structural responses is a primary consideration in the design of large flexible space structures such as Skylab. The Martin-Marietta Corporation, Denver Division under contract to Marshall Space Flight Center has developed the methodology and digital algorithm to determine the response of two elastic vehicles during the docking maneuver. The objective of this paper is to present the salient features of their analysis. More rigorous explanations and developments are contained in References 1 through 3.

Many authors have analyzed the docking maneuver in recent years. In general, their formulations were restricted to consideration of rigid vehicles Their docking analysis was performed in two steps; (1) calculation of the force-time histories at the docking mechanism between the vehicles and (2) application of the force-time histories from step 1 to elastic response analyses of the respective bodies. This procedure is referred to as the "two-step method." It was shown in Reference 4 that the two-step method can yield very conservative elastic responses for a highly flexible structure such a Skylab (Fig. 1). More realistic responses and loads are attainable by applying a "one-step method", wherein the effects of docking elastic rather than rigid vehicles are included. The Skylab Docking Response Analysis is a one-step method.

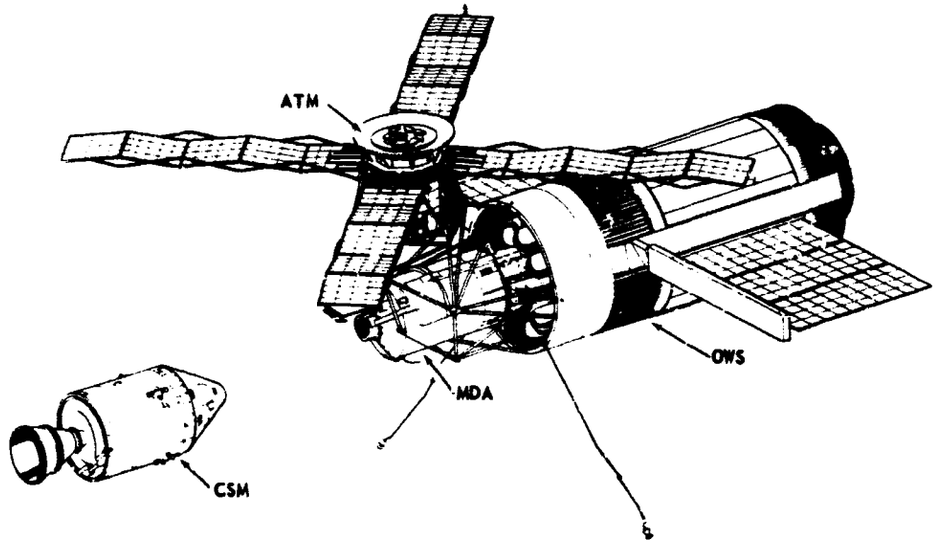

Figure 1. Skylab docking configuration.

PHYSICAL PROBLEM

The docking maneuver can be described as a controlled collision of two vehicles. The vehicles are considered to be initially in proximity such that collision is imminent. Each vehicle has a docking mechanism that serves to (1) dissipate the relative kinetic energy, (2) align the vehicles, and (3) bring the vehicles together and thus achieve a structural tie.

The Apollo probe/drogue docking system (Fig. 2) is utilized in the Skylab program. The docking probe is mounted on the command service module (CSM), and a drogue is mounted on the Skylab cluster at a radial docking port and also at the axial docking port. The CSM will be referred to as the chase vehicle, and the cluster will be referred to as the target vehicle.

The docking probe (Fig. 3) consists of a central body, probe head, pitch arms, tension links, hydraulic attenuators, capture latches, and support structure. The central body contains an inner cylinder that allows relative axial motion (10-inch stroke) with respect to an outer cylinder. The probe head is gimbal mounted on the inner cylinder and houses capture latches that engage the drogue apex.

The pitch arms and tension links act to align the vehicles and prevent any jack-knifing motion. The attenuators are fluid-displacement type units and are the primary sources of energy dissipation. The geometry of the probe and drogue is such that the potential points of contact are the probe head on the drogue surface and the pitch arms on the drogue lip.

ANALYTICAL DEVELOPMENT

During the course of the docking simulation, there are constraints that result from the geometry of the docking mechanism. As an example, consider the requirement that the motion of the probe head be constrained to be coincident with the motion of the drogue apex when the probe head is captured. Such conditions of constraint do not exist a priori, but are maintained by strong forces. It is a distinct advantage of the Lagrangian method [5] that allows one to formulate the equations of motion for a system with constraints without knowledge of the constraint forces. The constraint forces are implied by constraint equations. Now, the form of Lagrange's equations with auxiliary conditions of constraint are cited.

EQUATIONS OF MOTION

Consider the dynamical system to be characterized by discrete coordinates

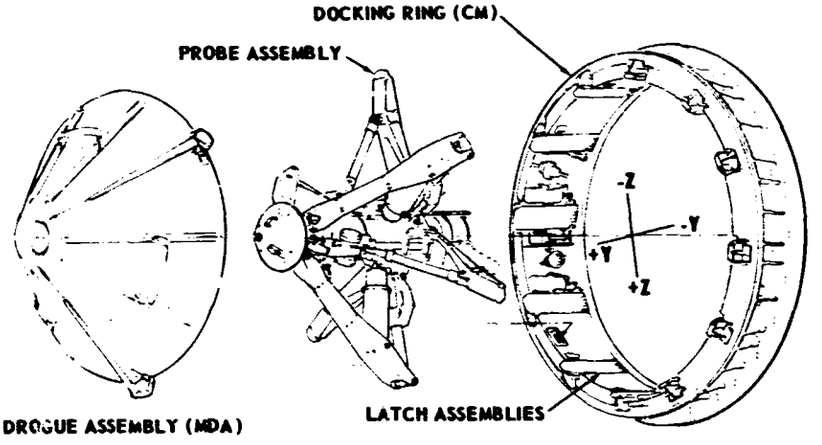

Figure 2. Docking system major assemblies.

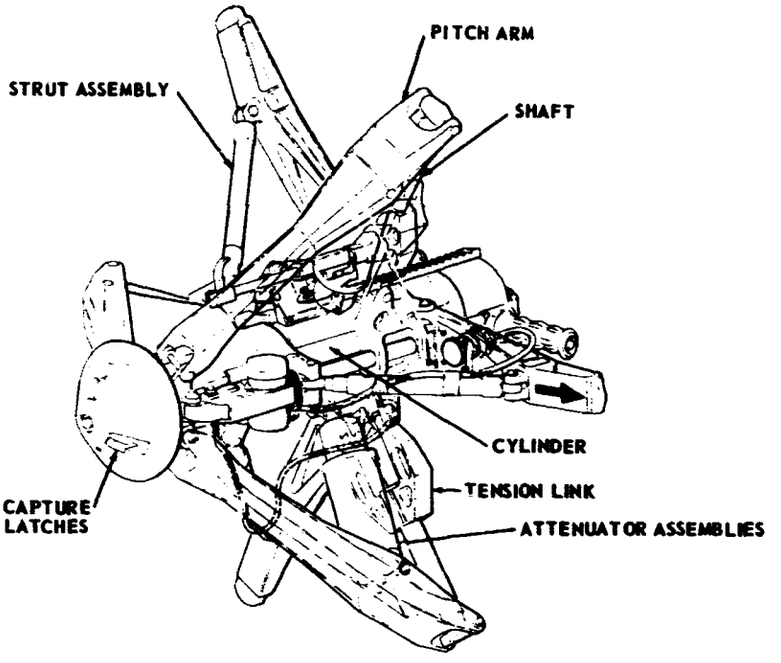

Figure 3. Probe docking system.

$$q_1, q_2, \ldots, q_n \quad . \tag{1}$$

The motion of the coordinates is restricted by kinematical conditions. These auxiliary conditions of constraint may be expressed in general form as

$$\bar{\phi}_s = \bar{\phi}_s (q_1, q_2, q_n, \dot{q}_1, \dot{q}_2, \dot{q}_n; t, dt) = 0 \quad . \tag{2}$$

$$s = 1, 2, \ldots, m$$

The Lagrange equations of motion can be expressed as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} = Q_k^* + \sum_{s=1}^{m} \bar{\lambda}_s \cdot \phi_s \quad . \tag{3}$$

In the above equation, T is the kinetic energy of the whole system. The generalized forces, Q_k^*, are associated with all impressed forces both conservative and nonconservative. The Lagrangian multiplier, $\bar{\lambda}_s$, is a measure of the microscopic violation of the constrant equation $\phi_s = \bar{0}$, and is equivalent to the forces required to maintain the constraint. The m equations of constraint coupled with the n differential equations of motion are sufficient to solve for the n unknowns, q_n; and for the m unknowns, $\bar{\lambda}_m$.

In general, it is more convenient to apply equations (2) and (3) in matrix form. To achieve a matrix form, consider a transformation matrix, $[\beta]$, where

$$\{u\} = [\beta]\{\dot{q}\} \quad . \tag{4}$$

The vector $\{u\}$ contains nonholonomic velocities (velocity components projected on a totaling coordinate system).

A matrix [P] is defined such that the kth row of [P] is

$$(\text{kth row of } [P]) = \{\dot{q}\}^T \left[\frac{\partial \beta}{\partial q_k}\right]^T \quad . \tag{5}$$

The auxiliary conditions of constraint (equation 2) are expressed in the form

$$\overline{\phi}_s = \sum_{k=1}^{n} \overline{b}_{sk}\,\dot{q}_k + \overline{\phi}_{st} = \overline{0} \quad . \tag{6}$$

The term $\overline{\phi}_{st}$ has been added to account for rheonomic constraints. The kinetic energy, T, may be expressed in terms of the nonholonomic velocities and a mass matrix [M], or

$$T = \frac{1}{2}\{u\}^T[M]\{u\} \quad . \tag{7}$$

With equations (4 through 7), equations (2) and (3) can now be written in matrix form as

$$[\beta]^T[M]\{\dot{u}\} + \left([\beta]^T - [P]\right)[M]\{u\} = \{Q^*\} + [b]^T\{\lambda\} \tag{8}$$

and

$$[b]\{\dot{q}\} + \{\phi_t\} = \{0\} \quad . \tag{9}$$

The matrix $[\beta]$ is a square matrix that is, in general, non-singular; then,

$$\{\dot{q}\} = [\beta]^{-1}\{u\} \quad . \tag{10}$$

Now, if equation (8) is pre-multiplied by $\left([\beta]^T\right)^{-1}$ and equation (10) is substituted into equation (9), the following is obtained

$$[M]\{\dot{u}\} + \left([\beta]^T\right)^{-1}\left([\dot{\beta}]^T - [P]\right)[M]\{u\} = \left([\beta]^T\right)^{-1}\left(\{Q^*\} + [b]^T\{\lambda\}\right) \tag{11}$$

and

$$[b][\beta]^{-1}\{u\} + \{\phi_t\} = \{0\} \quad . \tag{12}$$

Equation (11) represents n differential equations of motion and equation (12) represents the m equations of constraint. The derivation of the explicit forms of the matrices [P], $[\beta]$, and [b] are too lengthy to include here and can be found in Reference 2.

STATE VECTOR

The equations of motion and auxiliary equations that characterize the docking maneuver comprise a set of first order differential equations of the general form

$$\dot{Y}_i = f_i\,(Y_1, Y_2, \ldots Y_n, t) \tag{13}$$

where

$$i = 1, 2, \ldots n \quad .$$

The elements of the vector $\{Y\}$ completely define the state of the system at any time. The relationships between $\{Y\}$, $\{\dot{Y}\}$, and t are the equations of motion, constraint, and auxiliary equations that describe the behavior of the system. The elements of $\{Y\}$ are initially given; by evaluating $\{\dot{Y}\}$ numerically, inputs into an integration algorithm that serves to increment $\{Y\}$ are obtained.

Let us define the elements of the state vector $\{Y\}$.

$$\{Y\} = \begin{bmatrix} \{u_T\} \\ \{X_T\} \\ \{\gamma_T\} \\ \{u_C\} \\ \{X_C\} \\ \{\gamma_C\} \\ \{\rho\} \\ \{\xi_T\} \\ \{\xi_T\} \\ \{\xi_C\} \\ \{\xi_C\} \\ \{\delta_{Cs}\} \end{bmatrix} , \tag{14}$$

where

$$\{Y_T\} = \begin{bmatrix} u_T \\ v_T \\ w_T \\ \omega_{xT} \\ \omega_{yT} \\ \omega_{zT} \end{bmatrix} , \tag{15}$$

$$\{X_T\} = \begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix} , \tag{16}$$

and

$$\{\gamma_T\} = \begin{bmatrix} \gamma_{11T} \\ \gamma_{12T} \\ \gamma_{13T} \\ \gamma_{21T} \\ \gamma_{22T} \\ \gamma_{23T} \end{bmatrix} . \quad (17)$$

The parameters u_T, v_T, and w_T are compnents of translational velocity of the target vehicle's center of mass, referred to the target body-fixed system (Fig. 4). The components of angular velocity of the target body-fixed system are ω_{xT}, ω_{yT}, and ω_{zT}. The vector positioning the target vehicle's center of mass from the origin of the inertial frame and referred to the inertial frame has components X_T, Y_T, and Z_T. The orientation of the target vehicle's body axes with respect to the inertial system is given by the six direction cosines, γ_{ijT}. The vectors $\{u_C\}$, $\{x_C\}$, and $\{\gamma_C\}$ have corresponding definitions for the chase vehicle. The vector $\{\rho\}$ contains six parameters that describe the state of the probe assembly,

$$\{\rho\} = \begin{bmatrix} \rho_x \\ \alpha_y \\ \alpha_z \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} \quad (18)$$

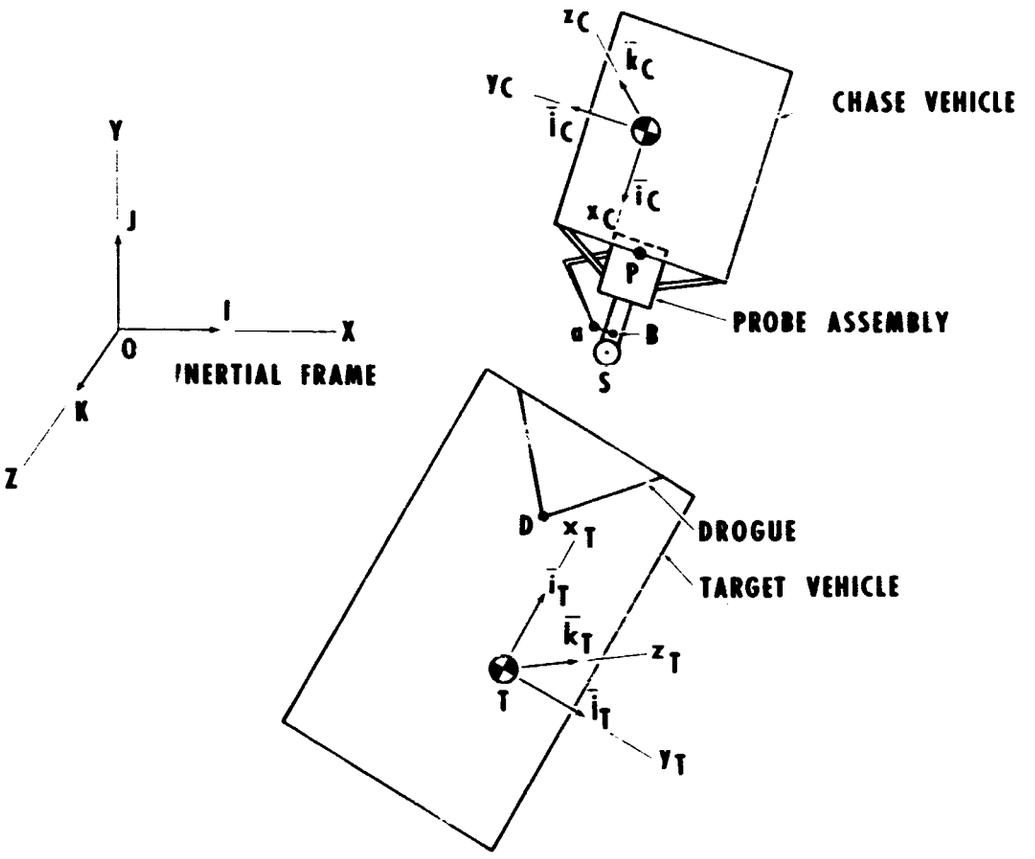

Figure 4. Coordinate systems.

The coordinate ρ_x describes the extension of the inner cylinder (Fig. 5). The angular deformation of the probe body resulting from elastic deflections of the probe support struts is described by the coordinates γ_y and γ_z. The coordinates (γ_1 = 1, 2, 3) are the angles included between the pitch arms and the probe central body.

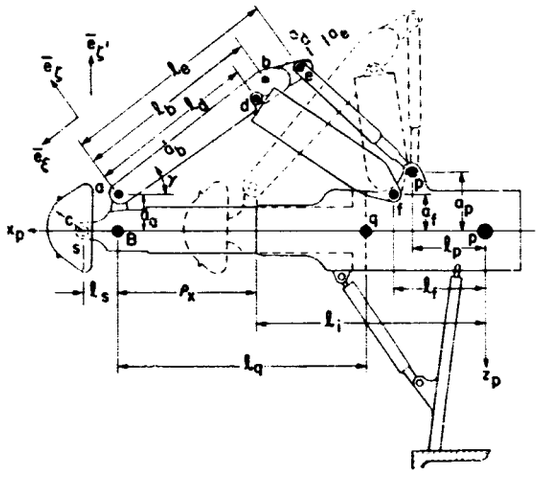

Figure 5. Probe mechanism geometry.

The elastic deflections of the target and chase vehicles are expressed as a superposition of normal free-free elastic modes of the respective vehicles. The vectors $\{\xi_T\}\{\xi_C\}$ contain the normal coordinates of the elastic modes of the target and chase vehicles, respectively. The vector $\{\delta_{Cs}\}$ contains the attitude control system parameters that are required to close the loop.

CONSTRAINT CRITERIA

There is a variety of combinations of constraint conditions that are to be implemented during the course of a motion simulation. The constraint criteria permit the mathematical model to simulate the various events that occur during the maneuvering in a representative manner. The motion during the maneuver is characterized by the bumping, sliding, breaking contact, and capture of the docking mechanisms.

A parameter ϕ_i is associated with each of the constraint conditions. The constraints are identified in Table 1.

TABLE 1. DESCRIPTION OF THE CONSTRAINTS [1]

Constraint Parameter	Description
ϕ_j, j=1,2,3	jth Tension Link Constraint $\phi_j = \lvert\overline{pe}\rvert_0 - \lvert\overline{pe}\rvert_j$; $\phi_j \geq 0$.
ϕ_4	Probe Head Contact Constraint $\phi_4 = -\overline{DS} \cdot \bar{e}_\alpha - C$; $\phi_4 \geq 0$.
ϕ_{j+4} j=1,2,3	jth Pitch Arm Contact Constraint $\phi_{j+4} = \overline{QR}_j \cdot \overline{m}_j - A_b$; $\phi_{j+4} \geq 0$
ϕ_8	Probe Head Capture Constraint $\phi_8 = \lvert\overline{DS}\rvert - C/\sin\alpha$; $\phi_8 \geq 0$

Because of the nature of the docking system, there are certain constraints that cannot physically occur at the same time. For example, the jth tension link cannot be in tension when the jth pitch arm is in contact with the drogue lip; i. e., ϕ_j and ϕ_{j+4} cannot simultaneously be equal to zero. A similar conflict exists between ϕ_4 and ϕ_8. This means that during the motion, if $\phi_i = 0$, then the corresponding constraint equation is a candidate to be used. When a conflict in constraint conditions occurs, the dominant condition is chosen.

There is a constraint force, λ, corresponding to each of the constraint equations. If a particular constraint equation is not in effect, its corresponding constraint force is zero. When particular constraint equations are in effect, the certain elements of the $\{\dot{Y}\}$ vector are solved to satisfy the constraint equations. The subvector $\{\dot{\rho}\}$ is that portion of $\{\dot{Y}\}$ that is committed to satisfy the constraint equations.

NUMERICAL EXAMPLE

A docking response analysis was accomplished considering the CSM docking to the axial port of the

cluster. The elastic modes of the cluster were included in one case and omitted in another. The CSM was considered rigid (inelastic) in both cases. For a given set of initial conditions, the docking force time histories were calculated and are plotted in Figures 6 and 7. The docking force is the resolution of the constraint forces to point D (Fig. 4). It is noted that the maximum force resulting from the elastic simulation is significantly less than that from the inelastic simulation.

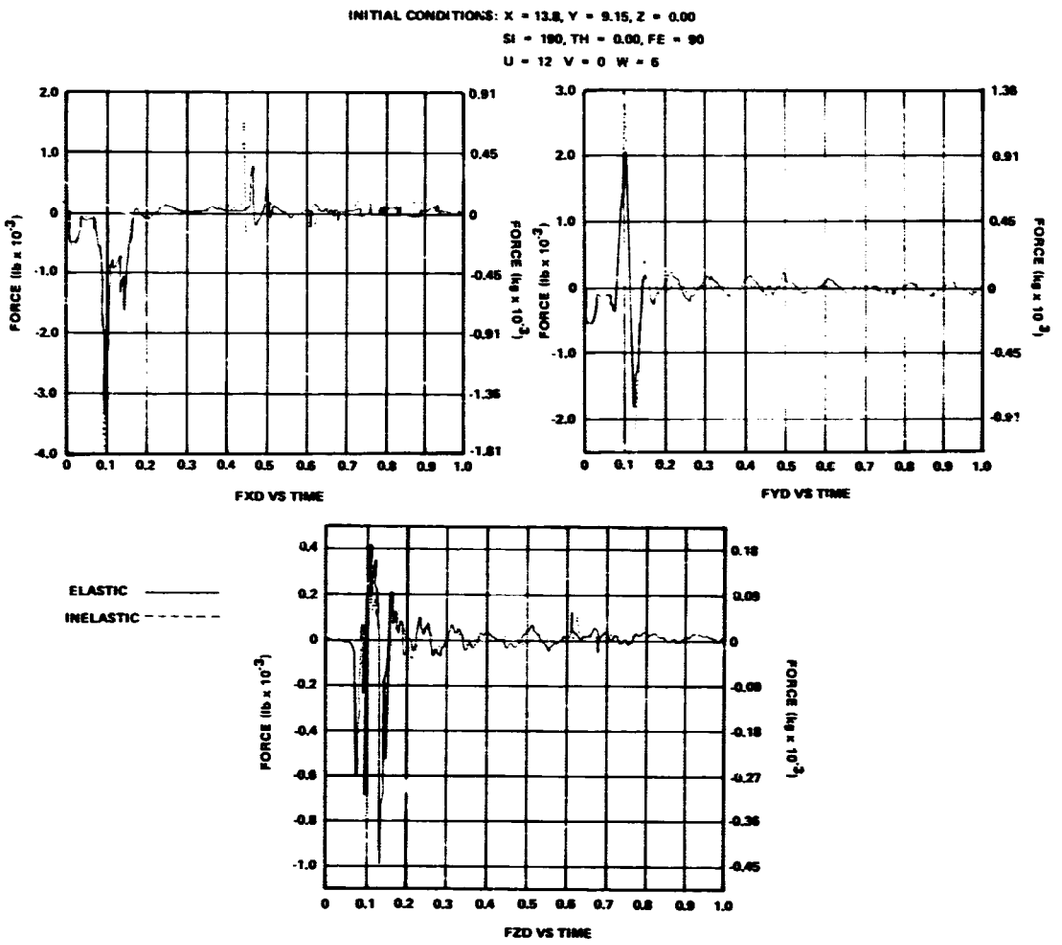
INITIAL CONDITIONS: X = 13.8, Y = 9.15, Z = 0.00
SI = 180, TH = 0.00, FE = 90
U = 12 V = 0 W = 6
FXD VS TIME
FYD VS TIME
FZD VS TIME
FORCE (lb x 10^-3)
FORCE (kg x 10^-3)
ELASTIC
INELASTIC

Figure 6. Docking force time history.

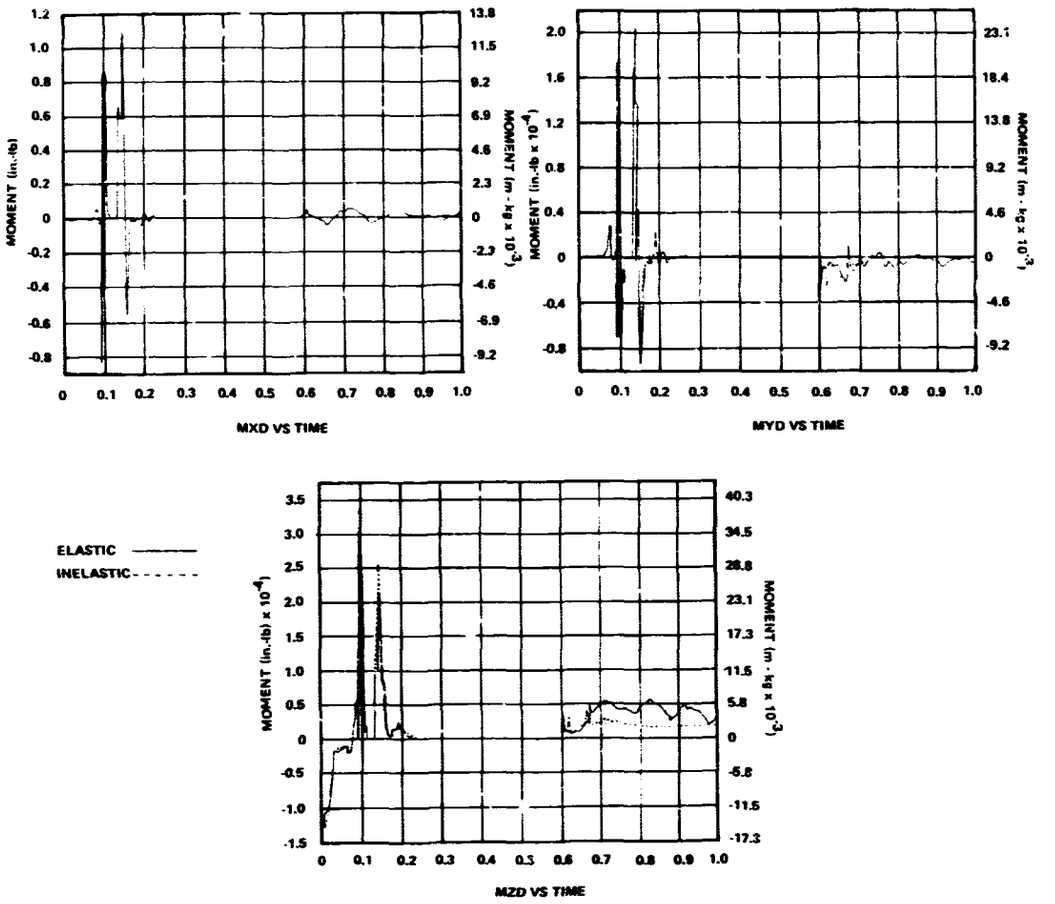

Figure 7. Docking force time history.

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